Es gilt:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{und damit auch} \quad \sum_{i=1}^{n} x_i = n\bar{x}.$$

Weiter gilt

$$\sum_{i=1}^{n} x_i\bar{x} = \bar{x}\sum_{i=1}^{n} x_i = n(\bar{x})^2$$

und

$$\sum_{i=1}^{n} (\bar{x})^2 = n(\bar{x})^2 \ .$$

$$\begin{aligned}
\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 &\overset{\text{binom. F.}}{=} \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2x_i\bar{x} + \bar{x}^2) \\
&= \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} x_i\bar{x} + \sum_{i=1}^{n}\bar{x}^2\right] \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2}{n}\bar{x}(n\bar{x}) + \frac{1}{n}n(\bar{x})^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2(\bar{x})^2 + (\bar{x})^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2 \ .
\end{aligned}$$